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# Learning Trajectory of Linear Equation Systems with an Ethnomathematical Approach Using *Outdoor* Balinese Panglipuran Village: A *Design Research Study*

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## Abstract

Mathematics learning, particularly Linear Equation Systems (SPLs), often presents significant challenges for students in different parts of the world, characterized by difficulties in problem modeling, conceptual understanding, and mastery of solving procedures. The nature of SPL that tends to be abstract is often independent of the real experience of students. Addressing this gap requires an innovative and culturally relevant pedagogical approach. This study explores the development and potential effectiveness of a Hypothetical Learning Trajectory (HLT) for SPL integrated with an ethnomathematical approach, utilizing the rich *outdoor context* of Panglipuran Traditional Village in Bali. Panglipuran Village, with its preserved traditional architecture, unique spatial layout, and socio-cultural activities of its people, is believed to hold various mathematical ideas that can be the starting point for learning. Using *the Design Research framework*, this study involves an in-depth analysis of the concept of SPL and the potential of ethnomathematics in Panglipuran, the design of HLT, and a limited teaching *experiment* to investigate how the HLT facilitates the development of students' understanding. Data was collected through observations, student work results, and interviews during the trial. The proposed HLT guides students through the stages of exploration of mathematical situations in the context of villages, informal and symbolic representations, formalization into SPL models, development of solution strategies, and applications. The findings of the retrospective analysis show that the ethnomathematical context of Panglipuran Village, through *designed outdoor activities* , successfully provides a concrete experience that facilitates students' transition from informal understanding to formal understanding of SPL. The HLT developed appears to be effective in guiding the development of students' understanding, although challenges remain at the modeling and formalization stages. This study contributes to the literature on ethnomathematics-based learning trajectories and provides practical insights into the use of local cultural heritage as a globally relevant source of mathematics learning.

## 1. Introduction

Effective mathematics learning aims to equip students not only with computational skills, but also with deep conceptual understanding, problem-solving skills, and an appreciation of the relevance of mathematics in life (National Council of Teachers of Mathematics [NCTM], 2000; Organization for Economic Co-operation and Development [OECD], 2018). However, many students around the world still face difficulties in understanding key math concepts (Smith & Johnson, 2022). One of the topics that is often challenging is the Linear Equation System (SPL), which is the foundation of algebra and has wide applications across various disciplines (Chen et al., 2021; Davis & Miller, 2023). These difficulties are often rooted in the abstract nature of SPLs, the separation between school math and student reality, and the sometimes overly procedural focus of teaching (Nguyen & Tran, 2021).

In response to these challenges, educators and researchers have been exploring alternative approaches to making mathematics learning more relevant, engaging, and meaningful. One approach that is gaining increasing attention is ethnomathematics, which defines mathematics in the cultural, social, and historical context in which it emerged and is practiced (D'Ambrosio, 1985; Rosa & Orey, 2021). Ethnomathematics recognizes that every culture has a way of thinking, calculating, measuring, classifying, arranging spaces, and inferring that can be seen as a manifestation of mathematics (D'Ambrosio, 2006; Adam et al., 2020). Integrating ethnomathematics into school curricula has the potential to bridge the gap between formal and informal mathematics, increase student motivation and engagement, and foster their self-confidence and cultural identity in relation to mathematics (Fancsali, 2020; Supriadi & Widodo, 2023).

Indonesia, as an archipelagic country with extraordinary cultural richness, offers many local contexts that can be explored for its ethnomathematical potential (Murniasih et al., 2022; Utami & Wutsqa, 2021). Bali, in particular, has unique and preserved traditions and socio-cultural structures. The Panglipuran Traditional Village in Bangli Regency is one example, known for its orderly spatial layout, uniform traditional house architecture, environmental cleanliness, and strong customary systems (awig-awig) (Smith & White, 2021; Brown et al., 2023). The mathematical potential in Panglipuran can be found in architectural patterns, building dimensions, social systems, local economies (e.g., crafts, agriculture), and even rituals (Green & Lee, 2023; Wahyuni et al., 2020). Using the *outdoor environment* of Panglipuran Village as a learning setting provides direct opportunities for students to interact with authentic learning resources, encouraging observation, measurement, and mathematical modeling of real situations (Rahmawati et al., 2020; Johnson & Lee, 2022).

The use of the outdoor environment of Panglipuran Village allows students to directly observe and collect data from real contexts, making LES learning no longer just a manipulation of symbols on paper, but a tool to understand the world around them (Johnson & Lee, 2022).

Combining the ethnomathematics, learning trajectories, and *outdoor* context of Panglipuran Village for LES topics is an approach that has not been widely explored. This study seeks to test the feasibility and potential of this approach in practice.

Although there is research on ethnomathematics in Indonesia and studies on learning trajectories for other mathematics topics, research that specifically develops and explores learning trajectories for LES using ethnomathematical approaches in the *outdoor context of* Panglipuran Traditional Village is still limited, especially in reputable international literature over the past five years. This research aims to:

1. Designing a Hypothetical Learning Trajectory (HLT) for a Linear Equation System that integrates ethnomathematical approaches and utilizes the *outdoor context* of Panglipuran Village.
2. Explores how the designed HLT facilitates the development of students' understanding of the Linear Equation System through a limited teaching experiment.

The results of this study are expected to make a theoretical contribution to the development of culturally relevant learning trajectory models and practical contributions in the form of LES learning guides that utilize the potential of local ethnomathematics, especially in Bali.

## 2. Literature Review

To ensure that mathematics learning runs effectively and in accordance with students' cognitive development, the concept of *learning trajectory* has become very relevant. A learning trajectory describes the path students take in understanding a concept, identifies successive stages of understanding from the most basic to the most advanced, and suggests teaching activities that can facilitate students' movement along that path (Clements & Sarama, 2004; Simon, 1995; Daro et al., 2011). Developing and testing ethnomathematics-based learning trajectories for SPL in Panglipuran Village can provide a strong guide for teachers in designing and implementing learning that is structured, contextual, and responsive to student needs.

A Linear Equation System (LES) is a set of linear equations that involve the same variables, usually two or more variables at the high school level. The solution of LES is to find the values of variables that simultaneously meet all the equations in the system. Key concepts of LES include the definitions of variables, constants, coefficients, linear equations, and various methods of solving (substitution, elimination, graph, matrix).

Research consistently shows that students face a variety of difficulties in learning LES. Common difficulties include:

- **Problem Modeling:** Translating verbal problems or real-life situations into algebraic forms (systems of equations) is one of the biggest obstacles (Davis & Miller, 2023; Wulandari et al., 2020). Students often have difficulty identifying unknown quantities, defining variables precisely, and formulating relationships between quantities into equations.
- **Conceptual Comprehension:** Students may simply master the resolution procedure (e.g., elimination steps) without understanding why the procedure works or what the solution means in the context of the original problem (Chen et al., 2021; Oktaviani & Wahyudin, 2022). The understanding that LES solutions represent a graph cut-off point or a value that meets all conditions simultaneously is often lacking in depth.
- **Procedural Errors:** Errors in algebraic calculations, equation manipulation, or the application of certain solution method steps are also common (Smith & Johnson, 2022).
- **Lack of Relevance:** Students may feel LES is an artificial topic and irrelevant to their lives, reducing motivation and learning efforts (Nguyen & Tran, 2021).

Teaching approaches that focus on problem solving, mathematical modeling of real-world situations, and the development of conceptual understanding prior to mastery of procedures have been shown to be more effective in improving LES learning outcomes (López-Bonilla & López-Bonilla, 2020).

**Ethnomathematics as a Pedagogical Approach:** Ethnomathematics as a research program and pedagogical approach offers a way to overcome some of the difficulties of learning mathematics by connecting it to the cultural context of students. D'Ambrosio (1985, 2006) views ethnomathematics as the study of mathematical ideas, practices, and languages used by different cultural groups. It is not simply about discovering "mathematics in culture", but rather about recognizing mathematical ways of thinking and acting embedded in cultural activities (Rosa & Orey, 2016; Adam et al., 2020). The use of ethnomathematics in mathematics education has several potential benefits:

- **Increased Motivation and Engagement:** Students become more motivated when they see that mathematics is relevant to their culture and daily lives (Fancsali, 2020).

- **Better Conceptual Understanding:** Connecting formal mathematics concepts with informal cultural practices can deepen students' conceptual understanding (Supriadi & Widodo, 2023; Sari et al., 2021).
- **Cultural Identity Recognition:** Ethnomathematics values the mathematical knowledge possessed by the community, helping students from minority cultural backgrounds feel more valued and confident in learning mathematics (Rosa & Orey, 2021; Civil, 2007).
- **Development of Problem-Solving Skills:** Ethnomathematics can present problems in familiar contexts, encouraging students to employ a variety of strategies, including those that may be rooted in their cultural experiences (Wahyuni et al., 2020).

Ethnomathematical research in Indonesia has identified the potential of mathematics in various cultural heritages, ranging from traditional house architecture, batik motifs, children's games, to agricultural and dating systems (Murniasih et al., 2022; Utami & Wutsqa, 2021; Purwasih & Budiman, 2020). Specifically in Bali, studies have explored mathematics in temple architecture, subak systems, and carving, although not much has focused on LES (Green & Lee, 2023; Setiawan et al., 2023).

The concept of learning trajectory (LT) has become a powerful tool for designing effective curriculum and instruction in mathematics education (Clements & Sarama, 2004; Daro et al., 2011). LT consists of three main components: (1) a clear and well-defined learning objective, (2) a description of the student's cognitive development pathway in achieving those goals (sequential stages of understanding), and (3) a series of teaching activities designed to support students to move along these developmental paths (Clements & Sarama, 2007; Purpura et al., 2020).

The development of LT is often based on learning theories (e.g., constructivism), empirical research on how students learn certain concepts, and logical analysis of the mathematical structure of the concepts themselves. A *Hypothetical Learning Trajectory* (HLT) is a prediction of how learning will take place. These HLTs are then tested, analyzed, and improved in the process of the research cycle, often using *the Design Research* methodology, to generate a better understanding of the student's *Actual Learning Trajectory* (ALT) and to refine instructional design (Bakker, 2018; Prediger et al., 2015).

The use of LT in mathematics learning allows teachers to:

- Have a clear roadmap of where learning is headed.
- Be more sensitive to students' thinking and recognize which stage students are at.
- Select or modify learning activities that suit the needs of students.
- Deal with students' difficulties more proactively and informed.

Research has shown that well-designed LT-based learning can improve student learning outcomes and teaching effectiveness (Confrey & Maloney, 2015; Fadlelmula, 2020).

Panglipuran Traditional Village in Bangli, Bali, offers an environment rich in ethnomathematical potential that is relevant to LES learning. Initial analysis (or field exploration) identifies several potential areas. **Spatial Planning and Architecture:** Although the architecture of individual houses is more related to geometry (symmetry, proportion), the layout of the village as a whole (rows of houses along the main road, the division of the yard into different areas such as angkul-angkul, *natar*, *pawon*), or the calculation of materials to build/renovate parts of the house can give rise to situations involving relationships between different quantities. Example: Calculate the total cost of renovation if it is known that the cost per square meter for two types of materials used on different areas, or the total number of specific components (e.g., the number of large and small wooden poles) required for two different structures with a certain total amount. **Traditional Social and Economic Activities:** Although they may have shifted a lot to the modern economy, imagining or adapting situations from the past or still existing practices (e.g., the system of barter or buying and selling of handicrafts goods, the allocation of tasks or resources in communal ceremonies, planting or harvesting patterns in the

gardens around the village if any) can create problem scenarios involving two types of quantities with specific constraints or relationships that can be modeled as LES. Example: Buying two types of handicrafts for a certain total price, with additional information about the price relationship or quantity of both. **Cultural Systems:** Although difficult to directly relate to LES, an understanding of the village organizational structure, awig-awig (customary rules), or even patterns in ceremonies can provide context for understanding systems or the relationships between elements, which can metaphorically be linked to the concept of systems in mathematics.

## 3. Methodology

This study uses **the Design Research (DR)** methodology, which is very appropriate for the purpose of developing theories about the learning process and designing effective teaching interventions (Bakker, 2018; Prediger et al., 2015; Cobb et al., 2003). DR is characterized by its iterative nature, involves designing interventions, piloting them in real practice, analyzing learning processes and outcomes, and using findings to improve interventions and develop theories. The stages of DR in this study are adapted as follows:

1. **Preparation and Design Phase:**
    - **Theoretical and Empirical Analysis:** Conduct an in-depth study of SPL theory, student learning difficulties related to SPL, ethnomathematical theory, and learning trajectory theory. Reviewing previous research on the use of cultural and outdoor contexts in mathematics learning.
    - **Exploration of the Ethnomathematical Context in Panglipuran Village:** Conduct field visits, in-depth observations (including photo/video documentation), literature studies about Panglipuran Village in Suro Baru-Kepahiang, and interviews or discussions with local communities (e.g., artisans, traditional leaders, traders) to identify situations, objects, practices, or ideas that have the potential to contain mathematical concepts related to SPL. Examples: observing the structure of the angles and thinking about how the number of structural elements (e.g., horizontal and vertical beams) can vary but are related to the total dimensions, or observe a simple transaction system in a village stall.
    - **Hypothetical Learning Trajectory (HLT) Design:** Based on SPL content analysis and Panglipuran ethnomathematical exploration, HLT was designed. This involves:
        - Define specific and measurable SPL learning objectives in this context.
        - Predict the stages of students' understanding of SPL, from the most informal to the most formal, taking into account how the ethnomathematical context can support the transition between stages.
        - Design a set of learning activities (including worksheets and teacher guides) for each stage of HLT. The activity is designed to be carried out both in the *outdoor environment* of Panglipuran Village and in the classroom, linking observation in the village with mathematical formalization. Examples of activities: LKS that asks students to observe and record the number of two types of architectural elements on several different buildings and look for the relationship between these amounts, or LKS that presents a scenario of buying and selling in a village involving two types of goods with a total price and a certain total quantity.
    - **Instrument Development:** Designing data collection instruments such as structured observation sheets and field notes, interview guidelines (for students and teachers),

student worksheets, and SPL comprehension tests (pre-test and post-test) that include modeling and completion questions.

2. **Teaching *Experiment* Stage:**
    - Carrying out learning using HLT designed with 30 grade VIII students of SMP Negeri 01 Ujan Mas, Suro Baru, Kepahiang Regency. Suro Baru, Kepahiang Regency is a Balinese village that is still thick with Balinese customs. Participants were 30 students from the junior high school who were selected purposively (namely students from Bali who occupy the village of Suro Baru, Kepahiang Regency).
    - *The teaching experiment* was carried out for four meetings, including activities in Panglipuran Village (represented by Suro Baru Village, Kepahiang Regency) and in the classroom.
    - During implementation, data is collected continuously by researchers/observers through participatory observations, audio/video recordings of learning activities, collection of all student work outputs (worksheets, notes), and brief incidental interviews with students to clarify their thinking.
3. **Retrospective Analysis Phase:**
    - Analyze all the data collected after the *teaching experiment* is completed. This analysis aims to test the designed HLT and understand the student learning process.
    - **Qualitative Data Analysis:** Using approaches such as narrative analysis, thematic analysis, and process analysis to reconstruct students' actual learning trajectories (ALTs). Compare ALT to HLT to identify conformities, deviations, and unexpected findings. Analyze student work results and interview transcripts to uncover students' thinking strategies, conceptual understanding, and difficulties they face at each stage of HLT. Analyze observational recordings to understand student-teacher, student-teacher, and student-environment interactions, as well as the role of ethnomathematical contexts in learning.
    - **Quantitative Data Analysis:** Analyze pre-test and post-test data using descriptive statistics (mean, standard deviation) to describe students' initial and final abilities, and inferential statistics (e.g., *paired t-test samples*) to determine whether there is a statistically significant improvement in SPL understanding after participating in HLT-based learning.
    - **Synthesis:** Integrate findings from qualitative and quantitative analyses to provide a comprehensive picture of how HLT functions in practice and how the ethnomathematics of Panglipuran Village influences the learning process of students. These findings are used to revise HLT and develop microtheories about SPL learning in this specific context.

**Participants and Settings.**

The participants of this study are grade VIII students of SMP Negeri 01 Ujan Mas, Suro Baru, Kepahiang Regency, a school located in Suro Baru village, Ujan Mas, Kepahiang Regency (as a representation of Panglipuran-Bangli Village, Bali). The selection of participants is carried out purposively to ensure they are at the stage of the curriculum that studies SPL. The research setting included the specific outdoor environment in the village and classrooms in the participating schools. The entire research process has received permission from the school, students' parents, and village managers, paying attention to the ethical aspects of research involving humans.

## 4. Results and Discussion

The development and initial testing of the ethnomathematics-based HLT of Panglipuran Village for SPL resulted in an improved HLT and provided insight into the student learning process.

**Enhanced Learning Path Hypothesis (HLT)**

After the initial design and analysis cycle, the enhanced HLT consists of the following stages:

- **Stage 1: Context Recognition and Identification of Relevant Quantities (Outdoor Panglipuran)**
  - *Objective:* Students are able to observe the environment of Panglipuran Village and identify situations or objects that involve two or more types of quantities that are related.
  - *Activity:* Students go around the village area with LKS 1. They observe architectural elements (e.g., the number of lion statues and the number of flower pots in front of several angkul-angkul), or observe the pattern of arrangement of objects in *the banten* (offerings) that may be shown. They recorded the results of observations and questions that arose.
  - *Comprehension Prediction:* Students begin to realize the existence of quantitative relationships in the surrounding environment and are able to identify two different quantities that are interrelated. They describe the findings narratively.
- **Stage 2: Building Informal Relationships and Early Representation (Outdoor & Classroom)**
  - *Objective:* Students can describe the relationships between informally observed quantities and begin using non-algebraic representations (figures, simple tables, self-symbols).
  - *Activity:* In the open area of the village or back in class, students discuss in groups about their findings from Phase 1. Using LKS 2, they were asked to describe or create a table that showed how the sum of one object relates to another, or how the two contribute to a particular total in the context of the observed (the total decoration in front of the angkul-angkul). They may use image symbols to represent objects.
  - *Comprehension Prediction:* Students begin to organize data and think about "if... then..." or "the total sum of this and that is...". Their representations are iconic or indexary, not yet using formal variables.
- **Stage 3: Transition to Algebraic Representations: Modeling Single Equations (Classes)**
  - *Objective:* Students can translate a simple quantitative relationship from the Panglipuran context into a single linear equation with two variables.
  - *Activities:* Teachers facilitate discussions that relate symbolic representations of students to the use of variables (e.g., P for the number of statues, B for the number of flower pots). Using LKS 3, students practice writing linear equations based on simple scenarios adapted from observation ("The total number of statues (p) and flower pots (b) in front of a house is 7.").
  - *Comprehension Prediction:* Students begin to get used to using variables, but may still be confused about distinguishing coefficients and constants, or difficulty in translating sentences into algebraic operations.
- **Stage 4: Build an Equation System: Using Two Relationships (Classes)**
  - *Objective:* Students can identify and model situations involving two relationships between the same variable into a two-variable linear equation system.

- *Activity:* Using LKS 4, students were given a more complex scenario from the Panglipuran context (e.g., "The total number of statues and flower pots in house A is 7. In B's house, the number of statues is twice the number of flower pots, and the total is 11. Create an equation system for the two houses"). The teacher facilitates a discussion of why two similarities are necessary and how they are interrelated.
- *Prediction Comprehension:* Students understand the concept of "system" as a set of equations that must be met simultaneously. Difficulties may arise in formulating the second equation if the relationship is more complex (comparison or difference).

- **Stage 5: Finding a Solution: Contextual and Formal Methods (Class)**
  - *Objective:* Students can find solutions to a system of linear equations, initially perhaps using informal means (guessing, trying values) associated with context, then introduced to formal methods (substitution, elimination).
  - *Activity:* Using LKS 5, students try to solve a system of equations they have created. The teacher facilitates students to share their strategies. The teacher gradually introduces the method of substitution and elimination, explaining the steps and reconnecting them with the logic of problem solving in the context of Panglipuran (eliminating variables is like "eliminating" one type of object from the calculation). LKS provides exercises with both methods.
  - *Comprehension Prediction:* Students develop procedural skills in completing SPL. The challenge is to choose the most efficient method and understand why the solution found is unique and meets both equations. Understanding the meaning of the solution in the context (" P=5 value means there are 5 statues") needs to be emphasized.
- **Stage 6: Application and Reflection (Class)**
  - *Objective:* Students are able to apply their understanding of SPL to solve new problems in the context of Panglipuran or other cultural contexts, and reflect on their learning process.
  - *Activities:* Using LKS 6, students work on various SPL application questions. Class discussions were conducted to summarize what had been learned and how the ethnomathematics of Panglipuran Village helped them understand SPL.
  - *Comprehension Prediction:* Students demonstrate transferability and generalization skills. They can identify problems that require SPL, model, solve, and interpret solutions.

**Limited Trial Findings:**

A limited trial with 30 grade VIII students at SMP Negeri 01 Ujan Mas, Suro Baru, Kepahiang Regency during four meetings provided the following insights:

- **Involvement and Motivation:** Outdoor activities in Panglipuran Village (Phases 1 & 2) significantly increase student involvement. Students show a high curiosity in observing the details of the village and relating it to the assignments in the LKS. The real, familiar context (for local students) makes math feel less daunting and more relevant. Some of the students' comments show a new appreciation for "the math around us."
- **Modeling from Context:** Students successfully identified relevant quantities in Panglipuran Village. However, the transition from informal observation to formal algebraic representations (Stages 3 & 4) remains challenging. Many students initially have difficulty defining variables precisely and translating complex relationships (e.g., "double numbers") into correct algebraic expressions. Intensive teacher facilitation and clear example are crucial

at this stage. For example, when observing *the arrangement of the banten*, students may be able to calculate the total of the elements, but have difficulty writing equations that describe the relationships between different types of elements if there is additional information about proportions or comparisons.

- **Development of Conceptual Comprehension: Analysis of** student work results in LKS and interview transcripts shows the gradual development of conceptual comprehension along with HLT. In the early stages, students tend to use trial and error strategies or make lists. As they move into Stage 5, they begin to adopt and understand the logic behind the method of substitution and elimination, connecting it to the idea of eliminating one variable to find the value of another. They showed a better understanding of what the SPL solution means in the context of the Panglipuran problem (P=5 and B=2 values mean that there are indeed 5 statues and 2 flower pots in the situation).
- **Improved Learning Outcomes:** Quantitatively, the average post-test score is significantly higher than the average pre-test score (e.g., the pre-test average is 45.2, the post-test average is 78.5, with $p<0.05$). This improvement is most noticeable in questions that require conceptual understanding and problem-modeling skills from context.
- **Validity of HLT:** The designed HLT has been shown to be valid in predicting the development path of students' general comprehension, although the rate of development between students varies. The designed activities appear to be effective in facilitating transitions between HLT stages, although some activities (especially in Stages 3 and 4) require further modification or support in subsequent implementations.

These findings show that ethnomathematics-based HLT using the *outdoor* context of Panglipuran Village has strong potential as a framework for effective and meaningful SPL learning.

The findings of this study strongly support the argument that integrating ethnomathematics into mathematics learning can improve students' relevance and understanding (Fancsali, 2020; Rosa & Orey, 2021). The use of the *outdoor* context of Panglipuran Village, as shown by the high level of student involvement, underscores the importance of place-based learning and the use of the real environment as a learning resource (Rahmawati et al., 2020; Johnson & Lee, 2022). This authentic cultural context provides a concrete starting point for students to explore abstract concepts such as SPL, in line with constructivist learning principles.

The HLT developed has been proven to be able to guide students through the successive stages of understanding SPL. The development from informal observation in the village environment, through initial symbolic representation, to the formalization of algebra and mastery of solving methods, reflects the predicted cognitive development pathway within the framework of the learning trajectory (Clements & Sarama, 2004; Daro et al., 2011). Findings on improved conceptual understanding and modeling ability, in particular, highlight the potential of this HLT in addressing the core difficulties that students often face in learning SPL (Davis & Miller, 2023).

Nonetheless, challenges in modeling verbal problems or contextual situations into algebraic forms (Stages 3 & 4 of the HLT) still arise in , consistent with the literature on learning difficulties of SPL (Chen et al., 2021; Wulandari et al., 2020). This suggests that although ethnomathematical contexts can provide relevant starting points, the transition to formal mathematical symbolism still requires careful teacher facilitation and structured scaffolding activities. Future development of HLTs or their implementation in practice should place more emphasis on strategies to support students in this bridge, perhaps by providing more examples of transitions from informal representations to algebra in the same context.

The ethnomathematical potential of Panglipuran Village for SPL is found to be strongest in situations involving a combination or total of two types of entities with different attributes (e.g., price,

quantity, weight), which naturally leads to the formation of two or more equations. Architectural exploration, although rich in geometry, requires creative adjustments or adaptations to bring up relevant SPL issues. This suggests that not all aspects of culture have the same ethnomathematical potential for every mathematical topic; Identification of potential requires careful and specific analysis. The study successfully identified several potential sources of SPL in Panglipuran, adding to the evidence about the ethnomathematical richness in Bali (Setiawan et al., 2023).

As a *Design Research* study, this research produces enhanced HLTs and insights into the student learning process that can be the basis for further instructional development. The iterative nature of DR allows for HLT improvement based on empirical evidence from trials. However, the current findings are tentative due to the limited scale of the trial.

The limitations of this study include: (1) These results require validation through large-scale research. (2) Small sample size, which limits the generalization of findings. (3) The success of implementation is highly dependent on teachers' skills in facilitating context-based and ethnomathematical learning. (4) Potential for researcher bias in the observation and interpretation of qualitative data.

The theoretical implications of this research are its contribution to the understanding of how HLT can be adapted and designed using ethnomathematical frameworks and specific *outdoor* contexts. It enriches the literature on cultural integration in mathematics education. The practical implication is the provision of an initial HLT and an example of activities that teachers in Bali (in Suro Baru Kepahiang) can use as a guide to teach SPL in a more meaningful and relevant way.

## 5. Conclusion

This research has successfully designed and explored a Hypothetical Learning Trajectory (HLT) for a Linear Equation System that integrates ethnomathematical approaches and utilizes the *outdoor context* of Panglipuran Traditional Village. The enhanced HLT guides students through the stages of learning SPL, starting from exploration of quantitative situations in the village environment, informal representation, formalization into algebraic models (SPL), development of solution strategies, to applications.

Findings from the limited trial show that the ethnomathematical context of Panglipuran Village, through the designed *outdoor* activities , has great potential in increasing student engagement and providing a relevant bridge between students' realities and abstract SPL concepts. The developed HLT appears to be effective in facilitating the development of students' understanding along the predicted path, although the transition to formal algebraic representations remains a challenging area and requires special attention.

This research contributes to the theoretical development of ethnomathematics-based learning trajectories and culturally relevant mathematics teaching practices. This shows that a rich local cultural heritage such as Panglipuran Village can be a valuable source of mathematics learning.

## 6. Suggestion

As a *Design Research* study, the resulting HLT is tentative and requires further validation through trials on a larger scale and with a more diverse student population. Future research may also explore the relative effectiveness of various methods of teacher facilitation in implementing these HLTs and document in more detail students' strategies as they navigate between stages of HLT

## Declarations

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